Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6675/01)

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June 2008
Further Pure Mathematics FP2
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\tanh x)) = \frac{\operatorname{sech}^2 x}{\tanh x}$	M1 A1
	$= \frac{1}{\sinh x \cosh x} = \frac{2}{\sinh 2x} = 2\operatorname{cosech} 2x$ (*)	M1 A1 (4)
		4
	Notes	
	1M1 Any valid differentiation attempt including $\ln(e^x - e^{-x}) - \ln(e^x + e^{-x})$	
	1A1 c.a.o. (o.e e.g. $\frac{\cosh x}{\sinh x} - \frac{\sinh x}{\cosh x}$)	
	2M1 Proceeding to a hyperbolic expression in $2x$	
	2A1 c.s.o.	

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2.	$8\left(\frac{e^x + e^{-x}}{2}\right) - 4\left(\frac{e^x - e^{-x}}{2}\right) = 13$	B1
	$4e^x + 4e^{-x} - 2e^x + 2e^{-x} = 13$	
	$2e^{2x} - 13e^x + 6 = 0$ (or equiv.)	M1 A1
	$(2e^x - 1)(e^x - 6) = 0$	
	$e^x = \frac{1}{2}, \quad e^x = 6$	M1 A1ft
	$x = \ln\frac{1}{2}$ (or $-\ln 2$), $\quad x = \ln 6$	A1 (6)
		6
	Notes	
	B1 Correctly substituting exponentials for all hyperbolics	
	1M1 To a three term quadratic in e^x	
	1A1 c.a.o. (o.e.)	
	2M1 Solving their equation to $e^x =$	
	2A1ft f.t. their equation.	
	3A1 c.a.o.	

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3.	$\int \frac{3}{\sqrt{x^2-9}}\,dx + \int \frac{x}{\sqrt{x^2-9}}\,dx$	B1	
	$= \left[3\operatorname{arcosh}\frac{x}{3} + \sqrt{x^2-9}\right]$	M1 A1 A1	
	$= \left[3\ln\left(\frac{x+\sqrt{x^2-9}}{(3)}\right) + \sqrt{x^2-9}\right]_5^6$		
	$= \left(3\ln\left(\frac{6+\sqrt{27}}{3}\right) + \sqrt{27}\right) - \left(3\ln\left(\frac{5+4}{3}\right) + 4\right)$	M1 A1	
	$= 3\ln\frac{6+\sqrt{27}}{9} + \sqrt{27} - 4 = 3\ln\frac{2+\sqrt{3}}{3} + 3\sqrt{3} - 4$ (*)	A1	(7)
	Notes		**7**
	B1 Correctly changing to an integrable form. **1M1** Complete attempt to integrate at least one bit. **1A1** One term correct **2A1** All correct **2DM1** Substituting limits in all. **Must have got first M1** **3A1** Correctly (no follow through) **4A1** c.s.o.		

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4.	(a) $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}$, At $x = \sqrt{2}$ $\frac{dy}{dx} = \frac{6}{3} = 2$	M1 A1, A1
	$y - \text{arsinh}\,(2\sqrt{2}) = 2(x - \sqrt{2})$	M1
	$y = 2x - 2\sqrt{2} + \ln\left(3 + 2\sqrt{2}\right)$ (*)	A1 (5)
	(b) $\frac{3a^2}{\sqrt{1+a^6}} = 2$ $9a^4 = 4(1+a^6)$	M1 A1
	$4a^6 - 9a^4 + 4 = 0$ $(a^2 - 2)(4a^4 - a^2 - 2) = 0$	A1
	$a^2 = \frac{1 \pm \sqrt{1+32}}{8}$ $a = \sqrt{\frac{1+\sqrt{33}}{8}} \approx 0.92$	M1 A1 (5)
		10
	Notes **(a)1M1** Attempt to differentiate need $\left(1+x^6\right)^{-\frac{1}{2}}$ at least **1A1** correct **2A1** c.a.o. **2M1** Substituting into straight line equation (linear). Must use $x = \sqrt{2}$ **3A1** c.s.o. **(b)1M1** Their derivative = their gradient (condone x throughout) **2M1= A mark cao, any form** **1A1** quartic cao **3M1** Solving their quartic to 'a' = **2A1** c.a.o. (a.w.r.t. 0.92 to 2dp)	

Question number	Scheme	Marks
5.	(a) $I_n = \int_0^{\pi} e^x \sin^n x\,dx = \left[e^x \sin^n x\right] - \int e^x n \sin^{n-1} x \cos x\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right] + n\int e^x(-\sin^n x + (n-1)\cos x \sin^{n-2} x \cos x)\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right]_0^{\pi} = 0$	B1
	$I_n = -n\int e^x \sin^n x\,dx + n(n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx$	M1
	$I_n = -nI_n + n(n-1)I_{n-2} - n(n-1)I_n \qquad I_n = \frac{n(n-1)}{n^2+1}I_{n-2}$ (*)	M1 A1 (8)
	(b) $I_4 = \frac{4\times 3}{17}I_2, \qquad = \frac{12}{17}\times\frac{2}{5}I_0$	M1, A1
	$I_0 = \int_0^{\pi} e^x\,dx = \left[e^x\right]_0^{\pi} = ...$, $\quad I_4 = \frac{24}{85}\left(e^{\pi}-1\right)$	M1, A1 (4)
		12

(a)1M1 Complete attempt to use parts once in the right direction need $\sin^{n-1} x$
1A1 cao
2M1 Attempt to use parts again with sensible choice of parts, not reversing. Need to be differentiating a product.
2A1 cao
1B1 both = 0 at some point. (doesn't need to be correct, must must =0)
3DM1 I_n = expressions in $\int e^x \sin^k x\,dx$ **Depends on 2nd M**
4DM1 Expresssion in I_n and I_{n-2} to $I_n =$. **Depends on 3rd M**
3A1 c.s.o.
(b)1M1 I_4 in terms of I_2
1A1 I_4 correctly in terms of I_0 [o.e.]
2M1 $\int e^x dx$
2A1 c.a.o for I_4 .

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6.	(a) $\int \cosh x \arctan(\sinh x)\,dx = \sinh x \arctan(\sinh x) - \int \sinh x \frac{\cosh x}{1+\sinh^2 x}dx$	M1 A1 A1
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)$ $(+C)$	M1 A1 (5)
	Or: ....$-\int \tanh x\,dx$	
	$= \sinh x \arctan(\sinh x) - \ln(\cosh x)$ $(+C)$ M1 A1	
	<u>Alternative:</u>	
	Let $t = \sinh x$, $\frac{dt}{dx} = \cosh x$, $\int \arctan t\,dt = t\arctan t - \int \frac{t}{1+t^2}dt$ M1 A1 A1	
	$= ......- \frac{1}{2}\ln(1+t^2)$ M1	
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)$ $(+C)$ (or equiv.) A1	
	(b) $\frac{1}{10}[\sinh x \arctan(\sinh x) - \ln(\cosh x)]_0^2 = ...,$ 0.34 (*)	M1, A1 (2)
		7
	(a) <u>Alternative:</u>	
	Let $\tan t = \sinh x$, $\sec^2 t \frac{dt}{dx} = \cosh x$, $\int t\sec^2 t\,dt = t\tan t - \int \tan t\,dt$ M1 A1 A1	
	$= ......- \ln(\sec t)$ M1	
	$= \sinh x \arctan(\sinh x) - \ln\sqrt{1+\sinh^2 x}$ $(+C)$ (or equiv.) A1	
	Notes **(a)1M1** Complete attempt to use parts **1A1** One term correct. **2A1** All correct. **2M1** All integration completed. Need a ln term. **3A1** c.a.o. (in x) o.e, any correct form, simplified or not **(b)1M1** Use of limits 0 and 2 and 1/10. **1A1** c.s.o.	

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7.	(a) $\frac{2x}{16} - \frac{2y}{9}\frac{dy}{dx} = 0$ $\quad \left[\frac{dx}{dt} = 4\sec t \tan t,\ \frac{dy}{dt} = 3\sec^2 t\right]$	M1 A1	
	$\frac{dy}{dx} = \frac{9x}{16y} = \frac{36\sec t}{48\tan t} = \frac{3}{4\sin t}$	M1 A1	
	$y - 3\tan t = \frac{-4\sin t}{3}(x - 4\sec t)$	M1	
	$4x\sin t + 3y = 25\tan t$ (*)	A1	(6)
	(b) Using $b^2 = a^2(e^2-1)$: $\quad ae = \sqrt{a^2+b^2} = 5 \quad$ or $\quad e = \frac{5}{4}$	M1 A1	
	P: $\ 4\sec t = 5 \qquad \cos t = \frac{4}{5}$	M1	
	Coordinates of P: $(4\sec t, 3\tan t) = \left(5, \frac{9}{4}\right)$	M1 A1	(5)
	(c) R: $\ x = \frac{25\tan t}{4\sin t} = \frac{125}{16}$	M1	
	Area of PRS: $\quad \frac{1}{2}(SR \times SP) = \frac{1}{2} \times \left(\frac{125}{16} - 5\right) \times \frac{9}{4} = \frac{405}{128}\ \left(= 3\frac{21}{128}\right)$	M1 A1	(3)
		14	
	Notes **(a)1M1** Differentitating **1A1** c.a.o. **2M1** $\frac{dy}{dx}$ in terms of t. **2A1** c.a.o. **3M1** Substituting gradient of **normal** into straight line equation. **3A1** c.s.o. **(b)1M1** Use of $b^2 = a^2(e^2-1)$ **1A1** c.a.o. for ae or for e **2M1** Using x coordinate of focus= x coordinate of P, to get single term $f(t)$= constant. (**Allow recovery in (c)**) **3M1** Substituting into P coordinates to a number for x and for y. **2A1** c.a.o. **(c)1M1** Attempt to find x coordinate of R. **2M1** Substituting into correct template i.e. ½ x |their R_x - their H_x| x their P_y **1A1** c.a.o. 3 s.f. or better.		

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8.	(a) $\dot{x} = 3 + 3\cos t \quad \dot{y} = 3\sin t$	B1
	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1+\cos t} = \frac{2\sin \frac{t}{2} \cos \frac{t}{2}}{2\cos^2 \frac{t}{2}} = \tan\frac{t}{2}$ (*)	M1 A1 (3)
	(b) $s = \int \sqrt{\dot{x}^2 + \dot{y}^2}\, dt = 3\sqrt{2}\int \sqrt{1+\cos t}\, dt$	M1 A1ft
	$= 6\int_0^t \cos\frac{t}{2} dt = 12\sin\frac{t}{2}$ (Limits or establish $C = 0$ for A1) (*)	M1 A1 (4)
	(c) $\tan\psi = \tan\frac{t}{2} \Rightarrow \psi = \frac{t}{2} \Rightarrow s = 12\sin\psi$	B1 (1)
	(d) Surface area $= \int_0^t 2\pi y\sqrt{\dot{x}^2 + \dot{y}^2}\, dt = 18\sqrt{2}\pi\int (1-\cos t)\sqrt{1+\cos t}\, dt$	M1 A1ft
	$= 72\pi\int \sin^2\frac{t}{2}\cos\frac{t}{2}\, dt$	M1
	$= \ldots\ldots\ldots\left(\frac{2}{3}\sin^3\frac{t}{2}\right)$	M1 A1
	But $\sin\frac{t}{2} = \frac{s}{12} = \frac{L}{12}$, so surface area $= \frac{144\pi}{3} \times \frac{L^3}{12^3} = \frac{\pi L^3}{36}$ (*)	M1 A1 (7)
	(a)1B1 both **1M1** Attempt at y'/x' **1A1** cso – on paper need to see half angles **(b)1M1** Attempt at arc length, integral formula **1A1** cao follow through on their x' and y' **one variable only** **2M1** Integrating **2A1** cso – on paper **(c) 1B1** cao **(d) 1M1** Attempt at Surface area, integral formula.Condone lack of 2π. **1A1** cao follow through on their x' and y' condone lack of 2π. **one variable only** **2DM1**Getting to integrable form condone lack of 2π. **Depends on previous M mark.** **3DM1**integrating condone lack of 2π. **Depends on previous M mark.** **2A1** cao **4DM1**Eliminating t to give expression in L only **Depends on previous M mark.** **3A1** cso – on paper.	